Mass of sphere$=\pi\int_0^a(a^4-x^4)dx=\frac{4}{5}\pi a^5$.

$\therefore \frac{4}{5}\pi a^5=3\pi(a^4-\frac{1}{5})$, $4a^5=15a^4-3$, which evidently has a root slightly less than 3.75.

In the solution given by Dr. Zerr in the November MONTHLY if the parts be added so as to give the mass of the sphere the result is not homogeneous in a and is therefore evidently wrong. In getting M the upper limit for θ should be $\cos^{-1}(1/r)$ and not $\cos^{-1}(1/a)$. For M_1 we must subtract $2M$ from the mass of the sphere.

75. Proposed by B. F. FINKEL, A.M.,M.Sc., Professor of Mathematics and Physics, Drury College, Springfield, Mo.

A particle P, is held in a bent tube by two forces directed towards two fixed points, H and S. Show that the equation of the tube is $PS.PH=k^2$, if the forces are μ/PS and μ/PH.

I. Solution by GEORGE R. DEANE, C. E., B. S., Professor of Mathematics, Missouri School of Mines, Rolla, Mo.

Put $PS=r_1$, $PH=r_2$. By the principle of virtual work, we have,

$$\frac{\mu}{r_1}\delta r_1+\frac{\mu}{r_2}\delta r_2=0.$$

Let $f(r_1, r_2)=0$ be the equation of the curve. Then

$$\frac{\partial f}{\partial r_1}\delta r_1+\frac{\partial f}{\partial r_2}\delta r_2=0.$$

Eliminating δr_1 and δr_2,

$$\frac{\dfrac{\partial f}{\partial r_1}}{\dfrac{\partial f}{\partial r_2}}=\frac{\dfrac{\mu}{r_1}}{\dfrac{\mu}{r_2}}.$$

Whence, $-\dfrac{dr_2}{dr_1}=\dfrac{r_2}{r_1}$, $r_1dr_2+r_2dr_1=0$, $r_1r_2=k^2$.

The general theorem of which this is a particular case, is given in Minchin's *Statics*, Vol. I., page 88.

II. Solution by WILLIAM HOOVER, A. M., Ph. D., Professor of Mathematics and Astronomy, Ohio State University, Athens, Ohio.

Let s=any arc of the tube, r, r'=the distances of the particle from the centers of force at any time t, m=the same absolute intensities of the forces, and β =the velocity of projection.

If S, S' be the radial forces attracting the particle, we will have

$$\frac{d^2s}{dt^2}=-S\frac{dr}{ds}-S'\frac{dr'}{ds}\ldots\ldots(1).$$

But $S=m/r$, $S'=m/r'$, and (1) becomes

$$\frac{d^2s}{dt^2}=-\frac{m}{r}\frac{dr}{ds}-\frac{m}{r'}\frac{dr'}{ds}\ldots\ldots(2).$$

Multiply by $2(ds/dt)$ and integrate ; then

$$\frac{ds^2}{dt^2}=-m\log r^2-m\log r'^2+C\ldots\ldots(3).$$

When $r=a$, $r'=a$, $\dfrac{ds}{dt}=\beta$; $\therefore$ $C=\beta^2-m^2\log\dfrac{1}{a^4}$, and (3) is

$$\beta^2=m^2\log\frac{1}{r^2r'^2}+\beta^2-m^2\log\frac{1}{a^4}\ \ldots\ (4),$$

or, $rr'=a^2\ldots\ldots(5)$, a lemniscate.

[Other solutions of this problem will appear in the next issue.]

DIOPHANTINE ANALYSIS.

73. Proposed by M. A. GRUBER, A. M., War Department, Washington, D. C.

Find integral values for x and y in $\begin{pmatrix}2x^2-y^2=\square\\2y^2-x^2=\square\end{pmatrix}$.

I. Solution by the PROPOSER.

$$2x^2-y^2=\square=a^2\ldots\ldots(1),\ 2y^2-x=\square=(2).$$

From (1), $y^2=2x^2-a^2$. Substituting this in (2), we have $3x^2-2a^2=b^2$. Whence $x=\frac{1}{3}\sqrt{[3(2a^2+b^2)]}$, and $y=\frac{1}{3}\sqrt{[3(a^2+2b^2)]}$.

As far as I know, the only method of rationalizing both radicals is to put $a=b$. Then $x=y=a=b$.

Accordingly, no *different* integral values can be found for x and y.

This problem is the key to Problem 62, "To find four squares in arithmetical progression."

The roots of the four squares would then be, respectively,

$$a,\quad \tfrac{1}{3}\sqrt{[3(2a^2+b^2)]},\quad \tfrac{1}{3}\sqrt{[8(a^2+2b^2)]},\quad b.$$

The common difference of the squares is $\frac{1}{3}(b^2-a^2)$.

According to the above solution, the roots of the four squares could not *all* be rational integers ; *one* of them, at least, must be a *surd*. It is evident, however, that an infinite number of sets of four squares can be found in which *two* of the roots are rational integers.